

Exercise 27

Use a linear approximation (or differentials) to estimate the given number.

$$e^{0.1}$$

Solution

Compute the derivative of $y = e^x$.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^x) \\ &= e^x\end{aligned}$$

Consequently, the differential of $y = e^x$ is

$$dy = e^x dx.$$

In order to estimate $e^{0.1}$, set $x = 0$ and $dx = 0.1$.

$$dy = e^0(0.1) = 0.1$$

Note that dy here is the vertical distance from the function's actual value at $x = 0$ to the linear approximation's value at $x = 0.1$.

$$e^{0.1} \approx e^0 + 0.1 = 1.1$$